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## Optimum Design of High Speed Prop-Rotors

by

Aditi Chattopadhyay  
Arizona State University  
Tempe, Arizona 85287-6106

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### Objectives

The objective of this research is to develop optimization procedures to provide design trends in high speed prop-rotors. The necessary disciplinary couplings are all considered within a closed loop optimization process. The procedures involve the consideration of blade aeroelastic, aerodynamic performance, structural and dynamic design requirements. Further, since the design involves consideration of several different objectives, multiobjective function formulation techniques are developed.

### Accomplishments

#### Multiobjective Formulation

This goal of the first part of this study is to develop multiobjective optimization formulation procedures for application in multidisciplinary design problems. Due to the fact that some of the optimization problems involve more than one design objective, the objective function formulation is more complicated. In most of the existing work, the individual objective functions are combined using weight factors in a linear fashion. Such methods are judgmental as the answer depends upon the weight factors which are often hard to justify. Two multiobjective function formulation techniques have been investigated and implemented. They are the Minimum Sum  $\beta$  (Min  $\Sigma\beta$ ) [1] and the Kreisselmeier-Steinhauser (K-S) function [2] approaches.

#### Optimization Implementation

A nonlinear programming method, as implemented in the numerical code CONMIN [3], is used for the optimization. CONMIN uses the method of feasible directions. In the optimization process, many evaluations of the objective function and constraints are required before

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convergence to an optimum design is obtained. Therefore, the process can become computationally expensive if exact analyses are performed for every function evaluation. Therefore the use of an approximate analysis is implemented in the calculations of both the objective functions and the constraints. The approximate analysis used for this study is the two point exponential procedure developed by Fadel et al. [4]

### Optimization Problems

Several multidisciplinary optimization problems involving the coupling of necessary disciplines that are crucial for tilting prop-rotor design have been developed. For example, the propulsive efficiency in high speed cruise (400 knots) and the hover figure of merit have been simultaneously maximized using both the Min  $\Sigma\beta$  and K-S function approaches. Constraints are imposed on the first natural frequency in hover and on the total blade weight. Both aerodynamic and structural design variables are used [5,6]. Next, an integrated structural and aeroelastic optimization procedure using a composite box beam model as the structural load carrying member has also been accomplished [7]. Other work that has been completed includes a minimum rotor drive system weight optimization and a minimum blade weight optimization.

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**DESIGN OF HIGH SPEED PROPROPOTORS USING  
MULTIOBJECTIVE OPTIMIZATION TECHNIQUES**

by

Thomas R. McCarthy

and

Aditi Chattopadhyay

Department of Aerospace and Mechanical Engineering  
Arizona State University  
Tempe, Arizona 85287 - 6106

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# **Design of High Speed Proprotors Using Multiobjective Optimization Techniques**

by

Thomas R. McCarthy<sup>†</sup>  
and  
Aditi Chattopadhyay<sup>††</sup>

Department of Aerospace and Mechanical Engineering  
Arizona State University  
Tempe, Arizona

## **Introduction**

Over the last few years, there has been a revival of interest in VTOL aircraft capable of operating in fixed wing as well as rotary wing mode. High speed rotorcraft designs, such as the tilting rotor configuration, pose an entirely new problem in the rotary wing field. The design goals for this class of aircraft include low downwash velocity in hover, good low speed maneuverability and cruise speeds of 350 - 500 knots<sup>1</sup>. Several new concepts<sup>2-5</sup> have recently been proposed to meet these design goals. Extensive research performed in this field have led to the XV-15 research aircraft and ultimately to the production of the V-22 Osprey tilting rotor for the US Navy.

The combined requirements of efficient high speed performance of a fixed wing aircraft and good helicopter-like hover characteristics complicates the design process of tilting high speed proprotor aircraft. It is necessary to maintain good aerodynamic efficiency in high speed axial flight without degrading hover efficiency. This often leads to conflicting design requirements. For example, improved efficiency in high speed

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<sup>†</sup> Graduate Research Assistant, Member AIAA, AHS

<sup>††</sup> Associate Professor, Member AIAA, AHS, ASME

cruise demands high drag divergence Mach numbers which are normally associated with thin airfoils. This however, reduces the hover figure of merit by reducing  $C_T/\sigma$ . Therefore, to maintain the required thrust ceiling in hover, the rotor solidity has to increase. Also as the forward speed increases, helical tip Mach number limitations caused by whirl flutter, require a reduction in the rotor rotational velocity. Introducing blade sweep can alleviate this problem by reducing the effective chordwise Mach number, which allows for higher speeds, without reducing the rotor RPM. Therefore the proper design of proprotor blades capable of achieving the design objectives must consider the right combination of airfoil thickness and blade sweep in addition to other aerodynamic variables such as planform and twist.

Several studies have been performed<sup>6-9</sup> to study design trade offs between the two flight modes. For example, Johnson et al.<sup>6</sup> performed a detailed study on the performance, maneuverability and stability of high speed tilting proprotor aircraft, including the XV-15 and V-22. Liu and McVeigh<sup>7</sup> recently studied the use of highly swept rotor blades for high speed tilt rotor use. However, formal optimization techniques were not used in these studies. Recently an effort was initiated by Chattopadhyay and Narayan<sup>8,9</sup> to develop formal multidisciplinary optimization procedures for the design of civil high speed tilting proprotor blades. The propulsive efficiency in axial flight was maximized with constraints on the figure of merit in hover, aeroelastic stability in cruise and other aerodynamic and structural design criteria. The purpose of the present paper is to formulate the optimum design problem of high speed proprotors using multiobjective optimization techniques with the integration of the necessary disciplines.

### Problem Definition

A integrated, multiobjective optimization procedure is developed for the design of high speed proprotors with the coupling of aerodynamic, dynamic, aeroelastic and

structural criteria. The objectives are to maximize propulsive efficiency in high speed cruise and rotor figure of merit in hover. Constraints are imposed on rotor blade aeroelastic stability in cruise and on total blade weight. Two different multiobjective formulation procedures, the Min  $\Sigma\beta$  and the K-S function approaches are used to formulate the two-objective optimization problem.

### Aerodynamic Model

The rotor studied is a wind tunnel model an existing advanced technology proprotor, which is a three bladed rotor with a rigid hub. Cubic variations are assumed for the chord ( $c$ ) and twist ( $\theta$ ) distributions to model the blade aerodynamics,

$$c(\bar{y}) = c_0 + c_1(\bar{y} - 0.75) + c_2(\bar{y} - 0.75)^2 + c_3(\bar{y} - 0.75)^3 \quad (1)$$

$$\theta(\bar{y}) = \theta_0 + \theta_1(\bar{y} - 0.75) + \theta_2(\bar{y} - 0.75)^2 + \theta_3(\bar{y} - 0.75)^3 \quad (2)$$

where  $\bar{y}$  denotes the nondimensional blade radius. Note that  $c_0$  represents the chord and  $\theta_0$  the twist at the 75 percent blade radius, respectively. A quadratic lifting line is used and is defined as follows.

$$x = f(y) = \varepsilon_1 y + \varepsilon_2 y^2 \quad (3)$$

where  $\varepsilon_1, \varepsilon_2$  are constants that determine the curvature, and are defined such that

$$|\varepsilon_i| \leq \zeta_i \quad (4)$$

where  $\zeta_i$  ( $i = 1, 2$ ) are prescribed bound for the curvature parameters. These bounds allow for either forward or backward in-plane curvature. When  $\varepsilon_1$  and  $\varepsilon_2$  are equal to zero the lifting line will be a straight line. The blade sweep, based upon this lifting line distribution, assumes the following form

$$\begin{aligned}\Lambda(\bar{y}) &= \frac{180}{\pi} \tan^{-1} \left( \frac{dx}{dy} \right) \\ &= \frac{180}{\pi} \tan^{-1} (\epsilon_1 + 2\epsilon_2 \bar{y})\end{aligned}\quad (5)$$

where  $\Lambda(\bar{y})$  is the sweep distribution, in degrees, defined to be positive aft of the straight lifting line.

### Structural Model

The structural model used for the problem consists of a simple two-celled box beam as shown in Fig. 1. The beam is considered to be the principal load carrying member of the prop rotor and the stiffness contributions from the honeycomb and the nonstructural or tuning weights are placed at the blade tip and are distributed along the planform. The total blade weight comprises the weight of the box beam, the skin, the honeycomb and the nonstructural weights. The wall thicknesses of the box beam are assumed to vary in proportion to the chord distribution.

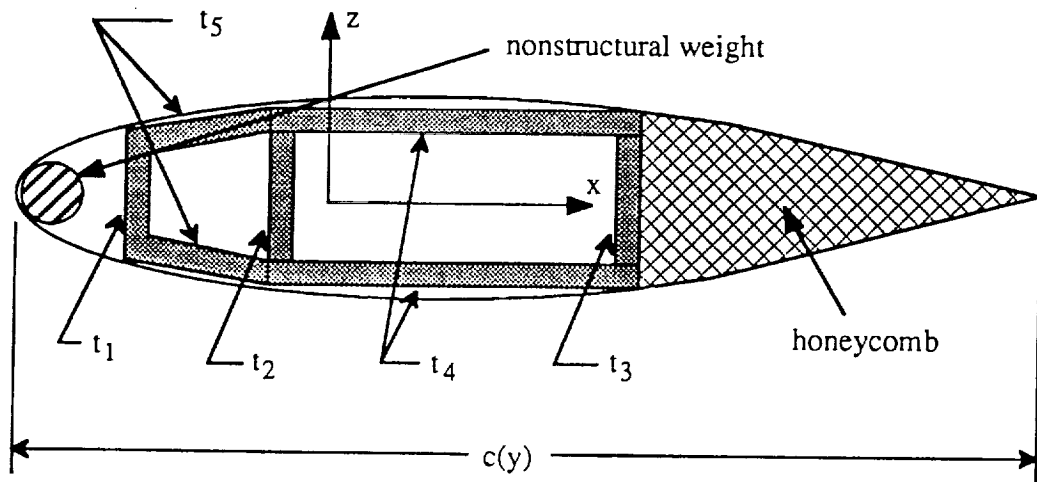


Figure 1 Double-celled box beam configuration

## Optimization

Objective Functions: The multiobjective optimization procedure is used to simultaneously maximize the rotor propulsive efficiency in high speed cruise and the hover figure of merit.

Design Variables: Both aerodynamic and structural design variables are used. The aerodynamic design variables include chord, twist and sweep distribution coefficients (Eqns. 1, 2 and 5). The structural design variables comprise roots values of the wall thicknesses of the two-cell box beam and the magnitudes of the nonstructural weights, at the tip, distributed spanwise.

Constraints: To avoid the possible occurrences of air and/or ground resonance associated with a soft inplane rotor, it is important to maintain the value of the lowest natural frequency in hover,  $f_1$ , above  $1/\text{rev}$ . Therefore the following constraint is imposed.

$$(i) \quad f_1 > 1/\text{rev} \quad (6)$$

It is important to impose aeroelastic stability constraints to prevent any degradation of the rotor stability in high speed cruise. This is all the more important when the blade mass and stiffness are altered during optimization. The stability constraints are expressed as follows.

$$(ii) \quad \alpha_k \leq -\nu_k \quad k = 1, 2, \dots, K \quad (7)$$

Where  $K$  represents the total number of modes considered, and  $\alpha_k$  is the real part of the stability root. The quantity  $\nu_k$  denotes the minimum allowable blade damping and is defined to be a small positive number. To avoid incorporation of weight penalties, after optimization, the total blade weight is constrained as follows.

$$(iv) \quad W \leq W_U \quad (8)$$



## Multiobjective Optimization

A typical optimization problem involving multiple objective functions can be mathematically posed as follows.

$$\begin{array}{llll}
 \text{Minimize} & F_k(\phi_n) & k = 1, 2, \dots, \text{NOBJ} & \text{(objective functions)} \\
 & & n = 1, 2, \dots, \text{NDV} & \\
 \text{Subject to} & & & \\
 & g_j(\phi_n) \leq 0 & j = 1, 2, \dots, \text{NCON} & \text{(inequality constraints)} \\
 & \phi_{nL} \leq \phi_n \leq \phi_{nU} & & \text{(side constraints)}
 \end{array}$$

where NOBJ denotes the number of objective functions, NDV is the number of design variables and NCON is the total number of constraints. The subscripts L and U denote lower and upper bounds, respectively, on the design variable  $\phi_n$ . A description of the multicriteria design objective formulation follows.

This study examines three multiobjective function formulation techniques that are less judgmental than the Pareto based weighting factors and are therefore more suited to large scale, highly nonlinear optimization problems that are associated with rotary wing design. The two multiobjective function techniques used are the Minimum Sum Beta ( $\text{Min } \Sigma\beta$ ) and the Kreisselmeier-Steinhausser (K-S) function approaches. A description of these methods follows.

Minimum Sum Beta ( $\text{Min } \Sigma\beta$ ): This method was first used by Weller et al.<sup>10</sup> to formulate a two objective function rotor vibration problem. Using these technique, pseudo-design variables that represent tolerances of the individual objective functions from prescribed tolerances are introduced. The objective function,  $\tilde{F}_1(\Phi)$ , is then defined

as a linear combination of these tolerances of each objective function to their specified target values as follows

$$\tilde{F}_1(\Phi) = \sum_{k=1}^{NOBJ} \beta_k \quad (9)$$

where  $\beta_k$  are pseudo design variables with properties such that the original objective functions  $F_k$  remain within a  $\beta_k$  tolerance of some prescribed values. This requirement introduces new constraints of the following form.

$$\frac{F_k - \tilde{F}_k}{\tilde{F}_k} \leq \beta_k \quad k = 1, 2, \dots, NOBJ \quad (10)$$

The quantities  $\tilde{F}_k$  are the prescribed target values of the individual objective functions  $F_k$ . Using the above formulation, as the objective function, and correspondingly the values of  $\beta_k$ , are reduced to zero, the values of the individual objective functions  $F_k$  are driven to their prescribed values,  $\tilde{F}_k$ . The design variables for the Min  $\Sigma\beta$  formulation comprise the original set of design variables and the pseudo design variables,  $\beta_k$ .

Kreisselmeier-Steinhauser (K-S) function: This technique was first utilized by Sobieski et al.<sup>11</sup> at the NASA Langley Research Center. The first step in formulating the objective function in this approach involves transformation of the original objective functions into reduced objective functions<sup>12</sup>. These reduced objective functions take the form

$$F_k^*(\Phi) = \frac{F_k(\Phi)}{F_{k_0}} - 1.0 - g_{\max} \leq 0 \quad k = 1, \dots, NOBJ \quad (11)$$

where  $F_{k_0}$  represents the value of  $F_k$  calculated at the beginning of each iteration. The quantity  $g_{\max}$  is the value of the largest constraint corresponding to the design variable vector  $\Phi$  and is held to be constant for each iteration. These reduced objective functions

are analogous to the previous constraints, and therefore a new constraint vector  $g_{2m}(\Phi)$  ( $m = 1, 2, \dots, M$ ) is introduced, where again  $M = \text{NCON} + \text{NOBJ}$ . The new objective function to be minimized is then defined, using the K-S function as follows:

$$\tilde{F}_2(\Phi) = f_{\max} + \frac{1}{\rho} \ln \sum_{m=1}^M e^{\rho(g_m(\Phi) - f_{\max})} \quad (12)$$

where  $f_{\max}$  is the largest constraint corresponding to the new constraint vector,  $g_{2m}(\Phi)$ , and in general is not equal to  $g_{\max}$ . The multiplier  $\rho$  can be considered analogous to a draw-down factor where  $\rho$  controls the distance from the surface of the K-S objective function to the surface of the maximum constraint function. When  $\rho$  is large the K-S function will closely follow the surface of the largest constraint function. When  $\rho$  is small, the K-S function will include contributions from all violated constraints. The design variable vector  $\Phi$  is identical to that used in the Min  $\Sigma\beta$  approach.

## Analysis

Dynamic, Aerodynamic and Aeroelastic Analyses The aerodynamic, dynamic and aeroelastic analysis of the high speed proprotor is performed using the code CAMRAD/JA<sup>13</sup>. The code has the capability of analyzing both helicopter and tilting rotor aircraft. Wind tunnel trim options are used as the reference blade is a wind tunnel blade model. In cruise, the blade is trimmed to specific rotor lift and drag coefficients using the rotor collective and cyclic pitch angles. A prescribed wake model, as implemented in CAMRAD/JA, is used to model the aerodynamics in hover and the rotor is trimmed to a specific value of the coefficient of power. In axial flight, the components of the induced velocity are negligible compared to the high forward speed of the rotor. Therefore, uniform inflow conditions are used to model the aerodynamics in this case.

The aeroelastic stability analysis in cruise is analyzed with a constant coefficient approximation.

Structural Analysis: The detailed structural analysis of the rotor blade is performed based upon the two-celled trapezoidal box beam using an inhouse code that was recently developed specifically for this application.

### Optimization Implementation

The optimization is performed by using the program CONMIN<sup>14</sup>. The program uses the method of feasible directions to solve nonlinear constrained optimization problems. To reduce the computational effort, an approximate analysis technique is used to compute the objective function and constraint values during iterations within the optimizer. For this problem the two-point exponential hybrid approximation technique<sup>15</sup> is used. This technique takes its name from the fact that the exponent used in the expansion is based upon gradient information from the previous design point. The technique is formulated as follows.

$$\hat{F}(\Phi) = F(\Phi_0) + \sum_{n=1}^{NDV} \left[ \left( \frac{\phi_n}{\phi_{0n}} \right)^{p_n} - 1.0 \right] \frac{\phi_{0n}}{p_n} \frac{\partial F(\Phi_0)}{\partial \phi_n} \quad (13)$$

where

$$p_n = \frac{\log \left\{ \frac{\left( \frac{\partial F(\Phi_1)}{\partial \phi_n} \right)}{\left( \frac{\partial F(\Phi_0)}{\partial \phi_n} \right)} \right\}}{\log \left( \frac{\partial \phi_{1n}}{\partial \phi_{0n}} \right)} + 1.0 \quad (14)$$

The quantity  $\Phi_1$  refers to the design variable vector from the previous iteration and the quantity  $\Phi_0$  denotes the current design vector. The exponent  $p_n$  can be considered as a “goodness of fit” parameter, which explicitly determines the trade-offs between traditional and reciprocal Taylor series based expansions (also known as a hybrid approximation technique). Details of this method can be found in Ref. 15.

## Results and Discussions

A wind tunnel model of an existing high speed propotor is used as a baseline design. The optimization for this problem is performed with a cruise velocity of 400 knots and a rotational velocity of  $\Omega = 375$  RPM (tip speed of 491 ft/s) in axial flight. The operating condition is 20,000 feet above sea level. In hover, a rotational velocity of  $\Omega = 570$  RPM (tip speed of 746 ft/s) is used at sea-level conditions. The high forward flight speed of 400 knots represents the target cruise value for high-speed rotorcraft. The tip speed is reduced in the airplane mode so that the helical tip Mach number stays below  $M_{dd}$  (the drag divergence Mach number). The rotor RPM in cruise (375) is selected after performing a parametric study on the effect of forward speed and rotor RPM on propulsive efficiency. A value of  $C_T/\sigma = 0.08$  is used to trim the blade in forward flight, and a value of  $C_P/\sigma = 0.0131$  is used to trim the blade in hover. The blade radius is 12.5

feet, and the blade is discretized into 10 segments ( $NSEG = 10$ ). For the  $\text{Min } \Sigma\beta$  approach 24 design variables are used (including the pseudo-design variables), and in the K-S function approach 22 design variables are used.

Some results, obtained to date, are summarized in Table 1 and Figs 2 and 3. Table 1 presents a summary of preliminary optimization results. From Table 1 and Fig. 2 it can be seen that it is possible to obtain substantial increases in both the hover figure of merit, (21.7 - 28.8 percent), and the axial propulsive efficiency,  $\eta_{ax}$  (24.6 - 41.3 percent) using both multiobjective formulation techniques. It is of interest to note that the mean chord (and correspondingly the blade solidity) is increased by 71 percent and 40 percent in the K-S function and  $\text{Min } \Sigma\beta$  approaches, respectively from the baseline value. Two possible explanations exist for this large increases in the rotor solidity. First, in order to satisfy the frequency constraint, the root chord is significantly increased to make the stiffnesses larger, which in turn increases the solidity (see Fig. 3). Secondly, since the hover figure of merit is being maximized,  $\sigma$  is being increased to increase the thrust margin of the rotor in hover.

Based on the previous experience, the above problem will be formulated with constraints on rotor solidity. The final paper will present results of the integrated aerodynamic/dynamic/aeroelastic optimization problem of high speed proprotors with additional design constraints. Design trade-off studies will also be performed by varying the flight conditions and the results of corresponding optimum blade designs will be presented.

Table 1 Summary of Preliminary Optimization Results

	Reference blade	Bounds		Optimum	
		lower	upper	Min $\Sigma\beta$	K-S
Objective Functions					
FOM	0.662	-	-	0.853	0.936
$\eta_{\text{ax}}$	0.647	-	-	0.787	0.805
Constraints					
W (lb)	194	-	194	167	173
$f_1$ (per rev)	0.812	1.00	-	1.01	1.34
$\alpha_1$	0.096	-	-0.001	-0.040	-1.529
$\alpha_2$	0.096	-	-0.001	-0.040	-1.529
$\alpha_3$	-0.697	-	-0.001	-0.732	-0.169
$\alpha_4$	-0.697	-	-0.001	-0.732	-0.169
$\alpha_5$	-2.431	-	-0.001	-2.443	-2.502
$\alpha_6$	-0.170	-	-0.001	-0.265	-0.073
$\beta_1$	0.150	0.001	0.200	0.006	-
$\beta_2$	0.150	0.001	0.200	0.010	-
Mean chord $c_e$ (ft)	1.48	-	-	2.07	2.52
Solidity $\sigma$	0.113	-	-	0.158	0.193
Trim $C_T/\sigma$	0.110	-	-	0.117	0.116

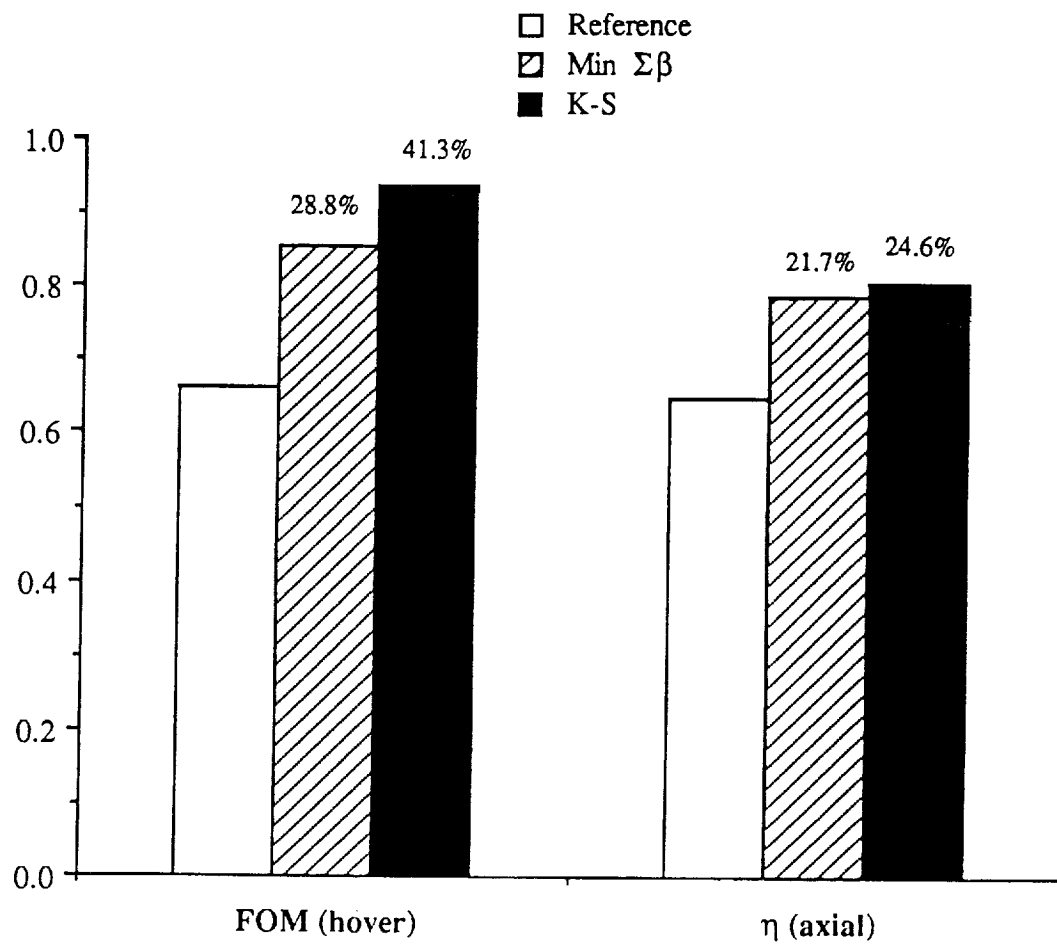


Figure 2 Comparison of individual objective functions



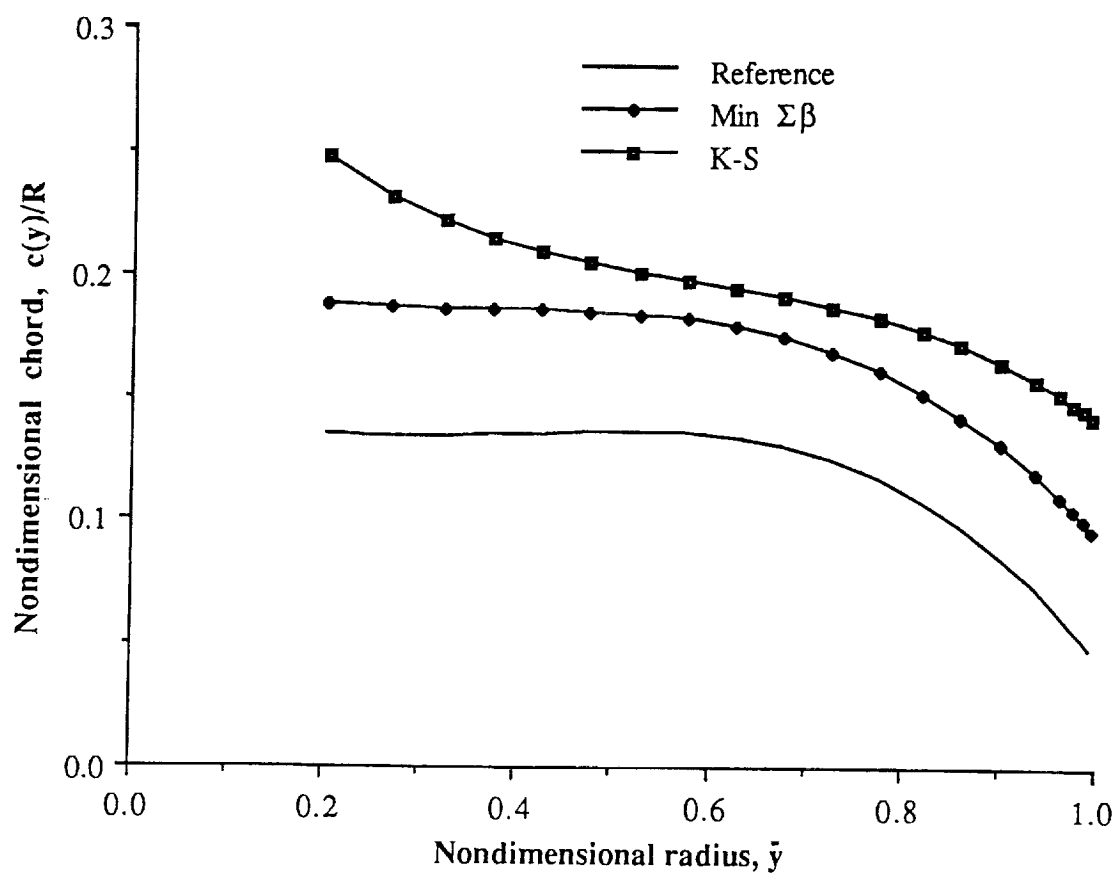


Figure 3 Chord distribution

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**Optimum Design of High Speed Prop Rotors Including the Coupling of  
Performance, Aeroelastic Stability and Structures**

by

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**Aditi Chattopadhyay, Thomas R. McCarthy  
Department of Mechanical and Aerospace Engineering  
Arizona State University  
Tempe, Arizona 85287-6106**

and

**John F. Madden, III  
Rotorcraft Technology Branch  
NASA Ames Research Center  
Moffett Field, California 94035**

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# OPTIMUM DESIGN OF HIGH SPEED PROP ROTORS INCLUDING THE COUPLING OF PERFORMANCE, AEROELASTIC STABILITY AND STRUCTURES

Aditi Chattopadhyay<sup>1</sup> and Thomas R. McCarthy<sup>2</sup>  
Department of Mechanical and Aerospace Engineering  
Arizona State University  
Tempe, Arizona  
and

John F. Madden, III<sup>3</sup>  
Rotorcraft Technology Branch  
NASA Ames Research Center  
Moffett Field, California

## Introduction

The primary design objectives of the 450 knot high speed civil prop-rotor are improved cruise propulsive efficiency and acceptable hover performance [1-5]. The rotor system should also be reasonably low weight and easy to operate. The vehicle design process is multidisciplinary in nature and involves a merging of several technical disciplines, such as aerodynamics, aeroelastic stability analysis, dynamics, structures and acoustics. For example, as speed increases, with the increase in rotor aerodynamic forces, the coupled motion of the prop-rotor and the wing elastic modes become unstable causing whirl flutter instability. Individual rotor blade stability is also important. Rotor aerodynamic performance requires ideal combination of planform, twist, airfoils and sweep to achieve maximum cruise propulsive efficiency while maintaining sufficient lifting capability and control properties in hover and in low-speeds. The complex rotor flow field associated with high tip velocities further complicates the aerodynamic analysis. In the helicopter mode, vibration can be a major source of problem and its alleviation will play an important role in the rotor blade design process. In the area of structures, the objective is to achieve maximum structural performance with minimum weight and cost requirements. Composite structures, combining the stiffness flexibility and bending-torsion coupling, are ideal for providing superior light-weight structure and Advanced Technology composite Blades (ATB) are currently being used in the XV-15 tiltrotor aircraft. Acoustics also plays an important role, as both external and internal noise generated by the vehicle are important design issues.

There also exists several conflicting design requirements. For example, cruise condition demands thin blade airfoils for high drag divergence Mach number ( $M_{dd}$ ), but thin airfoils have

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<sup>1</sup> Asst. Professor. Member, AHS, AIAA, ASME

<sup>2</sup> Graduate Research Assistant. Student member AHS, AIAA

<sup>3</sup> Aerospace Engineer. Member AHS

lower values of  $C_{l_{\max}}$  ( $C_l$  is the lift coefficient). This reduces the rotor figure of merit (FOM) by reducing  $C_T/\sigma$  ( $C_T$  is the thrust coefficient and  $\sigma$  is the thrust-weighted solidity). Therefore, to satisfy hover requirements, the rotor solidity has to increase. This can be eliminated by selecting airfoils with high  $C_{l_{\max}}$  and high  $L/D$  ratios at relatively low Mach numbers and low drag but high  $M_{dd}$  at high Mach numbers and low lift. Similarly, although airframe drag can be alleviated by sweeping the outboard sections, that operate at high Mach numbers, a swept rotor blade must be aeroelastically tailored to minimize the blade torsional and bending loads. The tradeoff between airfoil thickness and blade sweep angle therefore requires an indepth-study since they both influence aerodynamic and aeroelastic performance in hover and cruise.

The application of formal optimization procedures to tiltrotor design, therefore, seems appropriate as it can provide design trends while reducing the "man-in-the-loop" type of iterations. However, it is essential to integrate the necessary disciplinary couplings within the optimization process. Some initial investigations are due to Chattopadhyay and Narayan [7]. The purpose of the proposed paper is to enhance the state of the art by developing a multidisciplinary optimization procedure for high speed civil prop-rotors, including the couplings of aerodynamic performance, aeroelastic stability and structures in both high speed cruise and hover flight conditions.

## Problem Statement

An optimization procedure is developed for the design of high speed prop-rotors to be used in civil tiltrotor applications. The goal is to couple aerodynamic performance, aeroelastic stability and structural design requirements inside a closed-loop optimization procedure. The objective is to minimize the gross weight and maximize the propulsive efficiency in high speed cruise. Constraints are imposed on the rotor aeroelastic stability in both hover and cruise and rotor figure of merit in hover. Both structural and aerodynamic design variables are used.

## Multiobjective Optimization Formulation

Due to the fact that the optimization problem involves more than one design objective, the objective function formulation is complicated. In most of the existing work, the individual objective functions are combined using weight factors in a linear fashion [8]. Such methods are judgmental as the answer depends upon the weight factors which are often hard to justify. Also, in a rotary wing design environment, where complex nonlinear functions are involved, such methods are not well posed. Therefore an investigation is currently under way to evaluate the best formulation procedure appropriate for such applications. A Kreisselmeier-Steinhauser (K-S) function approach is used in this paper and a brief description of the approach follows.

**Kreisselmeier-Steinhauser (K-S) function approach:** The optimization problem can be stated as follows.

$$\begin{aligned}
& \text{Minimize} && F_k(\varphi_n) && k = 1, 2, \dots, \text{NOBJ} && \text{(objective functions)} \\
& && && n = 1, 2, \dots, \text{NDV} \\
& \text{subject to} && g_j(\varphi_n) \leq 0 && j = 1, 2, \dots, \text{NCON} && \text{(inequality constraints)} \\
& && \varphi_{nL} \leq \varphi_n \leq \varphi_{nU} && && \text{(side constraints)}
\end{aligned}$$

where NOBJ denotes the number of objective functions, NDV is the number of design variables and NCON is the total number of constraints. The subscripts L and U represent lower and upper bounds, respectively, on the design variable  $\varphi$ . Using the K-S function approach, the first step in formulating the objective function in this approach involves transformation of the original objective functions into reduced objective functions [9]. These reduced objective functions are of the form

$$F_k^*(\varphi) = \frac{F_k(\varphi)}{F_{k0}} - 1 - g_{\max} \leq 0 \quad k = 1, 2, \dots, \text{NOBJ} \quad (1)$$

where  $F_{k0}$  are the values of  $F_k$  calculated at the beginning of each iteration. The quantity  $g_{\max}$  is the value of the largest constraint corresponding to the design variable vector  $\varphi$  and assumed to be constant for this iteration. Because these reduced objective functions are analogous to the previous constraints, a new constraint vector  $g_j(\varphi)$ ,  $j = 1, 2, \dots, M$ , is introduced, where  $M = \text{NCON} + \text{NOBJ}$ . The new objective function to be minimized is then defined, using the K-S function as follows.

$$\tilde{F}(\varphi) = g_{\max} + \frac{1}{\rho} \log_e \sum_{m=1}^M e^{\rho(g_m(\varphi) - g_{\max})} \quad (2)$$

where the multiplier  $\rho$  is analogous to a draw-down factor controlling the distance from the surface of the K-S objective function to the surface of the maximum function value. The design variable vector  $\varphi$  is identical to that used in the Global Criteria approach. The method was found to provide faster convergence by Chattopadhyay and McCarthy [10] in a rotor blade optimization problem.

## Optimization Implementation

The basic algorithm used is the method of feasible directions as implemented in the optimization program CONMIN [11]. The optimization is to be coupled with a comprehensive analysis code. The blade is to be trimmed at each step of design optimization such that each feasible design (i.e., a design that satisfies all constraints) produced by the optimizer represents a trimmed configuration. Since the optimization process requires many evaluations of the objective function and constraints before an optimum design is obtained, the process can be very expensive if full analyses are made for each function evaluation. The objective function and constraints are therefore approximated by an approximation technique which is described below.

### Approximation Technique

A two-point exponential approximation [12] is proposed for approximation of the objective functions and the constraints. This technique takes its name from the fact that the exponent used in the expansion is based upon gradient information from the previous design point. This technique is formulated as follows.

$$\hat{F}(\Phi) = F(\Phi_0) + \sum_{n=1}^{NDV} \left[ \left( \frac{\phi_n}{\phi_{0n}} \right)^{p_n} - 1.0 \right] \frac{\phi_{0n}}{p_n} \frac{\partial F(\Phi_0)}{\partial \phi_n} \quad (3)$$

where

$$p_n = \frac{\log \left\{ \frac{\left( \frac{\partial F(\Phi_1)}{\partial \phi_n} \right)}{\left( \frac{\partial F(\Phi_0)}{\partial \phi_n} \right)} \right\}}{\log \left( \frac{\phi_{1n}}{\phi_{0n}} \right)} + 1.0 \quad (4)$$

The quantity  $\Phi_1$  refers to the design variable vector from the previous iteration and the quantity  $\Phi_0$  denotes the current design vector. A similar expression is obtained for the constraint vector. The exponent  $p_n$  can be considered as a “goodness of fit” parameter, which explicitly determines the trade-offs between traditional and reciprocal Taylor series based expansions (also known as a



hybrid approximation technique). It can be seen from Eqn. 4 that in the limiting case of  $p_n = 1$ , the expansion is identical to the traditional first order Taylor series, and when  $p_n = -1$ , the two-point exponential approximation equates to the reciprocal expansion form. The exponent is then defined to lie within this interval, such that if  $p_n > 1$ , it is set identically equal to one, and if  $p_n < -1$ , it is set equal to -1.

## Analysis

The aerodynamic, dynamic and aeroelastic analysis is performed using the comprehensive analysis code CAMRAD/JA [13]. The structural analysis is performed using a code developed in-house.

## Blade Modeling

The formulation and modeling assumptions used in the integrated optimization problem are described in this section.

### Aerodynamic Model

The aerodynamic lifting line offset, from reference axis, is based on a cubic model (Figure 1) with in-plane curvature as follows.

$$x_{ac}(\bar{y}) = \epsilon_0 + \epsilon_1 \bar{y} + \epsilon_2 \bar{y}^2 + \epsilon_3 \bar{y}^3 \quad (5)$$

where  $\epsilon_0, \epsilon_1, \epsilon_2$  and  $\epsilon_3$  are constants that determine the curvature and  $\bar{y}$  is the nondimensional radial location. These curvature parameters,  $\epsilon_i$  ( $i = 0 - 3$ ), are defined such that

$$|\epsilon_i| \leq \zeta_i \quad (6)$$

where  $\zeta_i$  are prescribed bounds for the curvature parameters. These bounds allow for either forward or backward in-plane curvature. When these parameters are equal to zero, the lifting line will be a straight line. Based upon this quadratic lifting line, the sweep variation, in degrees, is calculated as follows.

$$\Lambda(\bar{y}) = 180 (\epsilon_1 + 2\epsilon_2 \bar{y} + 3\epsilon_3 \bar{y}^2) / \pi \quad (7)$$

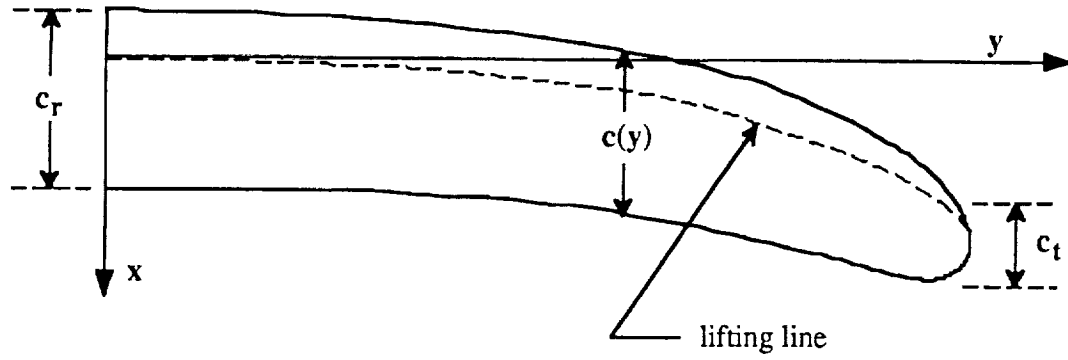


Figure 1 Blade model

The twist angle of attack,  $\theta(\bar{y})$ , is defined to have the following spanwise variation.

$$\theta(\bar{y}) = \theta_{\text{ideal}} + \theta_{\text{perturbation}} \quad (8)$$

where

$$\theta_{\text{ideal}} = \tan^{-1} \left( \frac{V_{\infty}}{\Omega \bar{y}} \right) \quad (9)$$

$$\theta_{\text{perturbation}}(\bar{y}) = \theta_1 (\bar{y} - 0.75) + \theta_2 (\bar{y} - 0.75)^2 + \theta_3 (\bar{y} - 0.75)^3 \quad (10)$$

and  $V_{\infty}$  represent the free stream velocity. It is important to note that twist plays an important role in both cruise and hover flight conditions and at high Mach numbers the twist has a significant effect on the cruise efficiency. Therefore, the above cubic distribution is chosen to provide the optimizer with more flexibility by using the coefficients  $\theta_1 - \theta_3$  as design variables.

## Structural Model

The load carrying structural member is a single cell composite box beam (Figure 2).

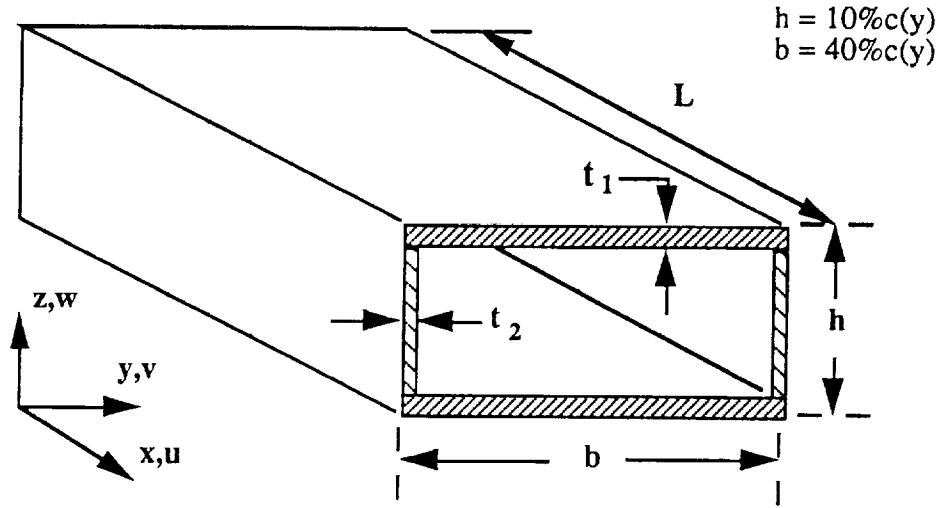


Figure 2 Single cell composite box beam

The beam is modeled with unequal vertical and horizontal wall thicknesses and the beam cross section is described by stretching, bending, twisting, shearing and torsion related warping. The box beam walls are made of layers of laminated orthotropic composite plies. A symmetric ply arrangement of  $(90^\circ/45^\circ/0^\circ/-45^\circ)_s$  is used for the horizontal walls and a symmetric arrangement of  $(0^\circ/\pm 45^\circ/0^\circ)_s$  is used for the vertical walls. The spanwise thickness distribution,  $t(y)$ , for each ply is based on the following spanwise distribution.

$$t(\bar{y}) = t_0 + t_1 \bar{y} + t_2 \frac{1}{\bar{y}} + t_3 \frac{1}{\bar{y}^2} \quad (11)$$

where  $t_0 - t_3$  are coefficients that determine the thickness distribution. The total thickness of an individual ply with orientation  $\psi$  is calculated as follows

$$t_\psi(\bar{y}) = n_\psi t(\bar{y}) \quad (12)$$

where  $t_\psi$  is the total thickness and  $n_\psi$  is the total number of plies with orientation  $\psi$ . The stiffnesses, as required by CAMRAD/JA, are calculated based on the formulation developed by Smith and Chopra [14].

Non structural tuning weights,  $w(\bar{y})$ , are placed at the leading edge and the following cubic spanwise distribution is assumed.

$$w(\bar{y}) = w_0 + w_1 \bar{y} + w_2 \bar{y}^2 + w_3 \bar{y}^3 \quad (13)$$

where  $w_0 - w_3$  are the coefficients describing the weight distribution. The total blade weight comprises the weight of the box beam, the tuning weights, the skin and the honeycomb weight (used in the trailing edge section).

**Objective function, design variables and constraints:** The objective function to be minimized is the total weight which comprises the drive system weight and the total weight of the three blades as shown below.

$$W = W_{\text{drive}} + W_{\text{blade}} \quad (14)$$

where

$$W_{\text{drive}} = W_{\text{engine}} + W_{\text{trans}} + W_{\text{fuel}} \quad (15)$$

and the individual weights are based upon the following empirical formulae.

$$W_{\text{engine}} = \eta_{e\Omega}(\text{SHP})(0.12) + 175 \quad (16)$$

$$W_{\text{trans}} = 300 \left( 1.1 \frac{\text{SHP}}{\text{RPM}} \right)^{0.8} \quad (17)$$

$$W_{\text{fuel}} = 1.5(\text{SFC})(\text{SHP}) \quad (18)$$

In the above equations, SFC is the specific fuel consumption,  $\eta_{e\Omega}$  is an empirical efficiency parameter, RPM is the rotational velocity of the rotor and SHP is the shaft horse power. The blade weight,  $W_{\text{blade}}$ , includes the box beam, the skin, the honeycomb and the non structural or tuning weights, placed at the leading edge. Both structural and aerodynamic design variables are used during the optimization. Following is a summary of the design variables.

- (i) thickness distribution coefficients,  $t_1 - t_3$ .
- (ii) number of plies with orientation  $\phi$ ,  $n_{\phi_i}$  ( $i = 1-5$ )
- (iii) non structural weight distribution coefficients,  $w_0 - w_3$
- (iv) coefficients of sweep distribution,  $\epsilon_0 - \epsilon_3$
- (iv) coefficients of twist distribution,  $\theta_1 - \theta_3$

To ensure aeroelastic stability in both hover and high speed cruise, constraints are imposed on the real part of the stability root as follows.

$$\alpha_{ki} \leq 0 \quad \begin{array}{l} k = 1, 2, \dots, \text{NMOD} \\ i = 1, 2 \quad (1 - \text{hover}, 2 - \text{axial}) \end{array} \quad (19)$$

where NMOD represents the total number of modes constrained. Also a lower bound constraint is imposed on the hover figure of merit FM as follows.

$$\text{FM} \geq \text{FM}_{\text{allow}} \quad (20)$$

where  $\text{FM}_{\text{allow}}$  is the required figure of merit in hover.

## Results

Some preliminary results obtained are presented in this section. The objective function used is minimizing the blade weight increment from reference to the composite model. Constraints are imposed on the rotor aeroelastic stability in hover and cruise. The rotor used, as a reference, is the XV-15 Advanced Technology Blade which is three bladed rotor with a rigid hub and zero sweep. The blade radius is 12.5 feet. The rotor was designed for a cruise velocity of 300 knots. The optimization is performed with both multi and single objective function formulation techniques. The program CAMRAD/JA is used for the aerodynamic, dynamic and aeroelastic analysis. The blade is discretized into 51 segments structural stations and 21 aerodynamic stations. Using a wind tunnel trim option, in each case, the optimum rotor is trimmed to the same  $C_T$  as the baseline or the reference rotor such that the same lifting capability is maintained after optimization. The optimization is performed at a cruise velocity of 400 knots at an altitude of 25,000 feet above sea level. In hover, a rotational velocity of  $\Omega = 570$  RPM (tip speed of 746 ft/s) is used at sea-level conditions. An uniform inflow model is used for both hover and axial analysis. In hover the rotor is trimmed to a design  $C_T/\sigma = 0.13$  and an  $L/D$  value of 5.3 is used in cruise. The material used for the composite box beam is T300/5208 Graphite Epoxy with a volume fraction of 70 percent. The aeroelastic stability of the reference blade is significantly improved as indicated in Figs. 4 and 5 for hover and cruise, respectively.

The final paper will contain results of the comprehensive optimization problem using total vehicle weight and high speed cruise propulsive efficiency as objective functions. The results of

the optimization, performed at a cruise velocity of 400 knots, will be compared against the baseline XV-15 ATB blade which has never been tested at such speeds.

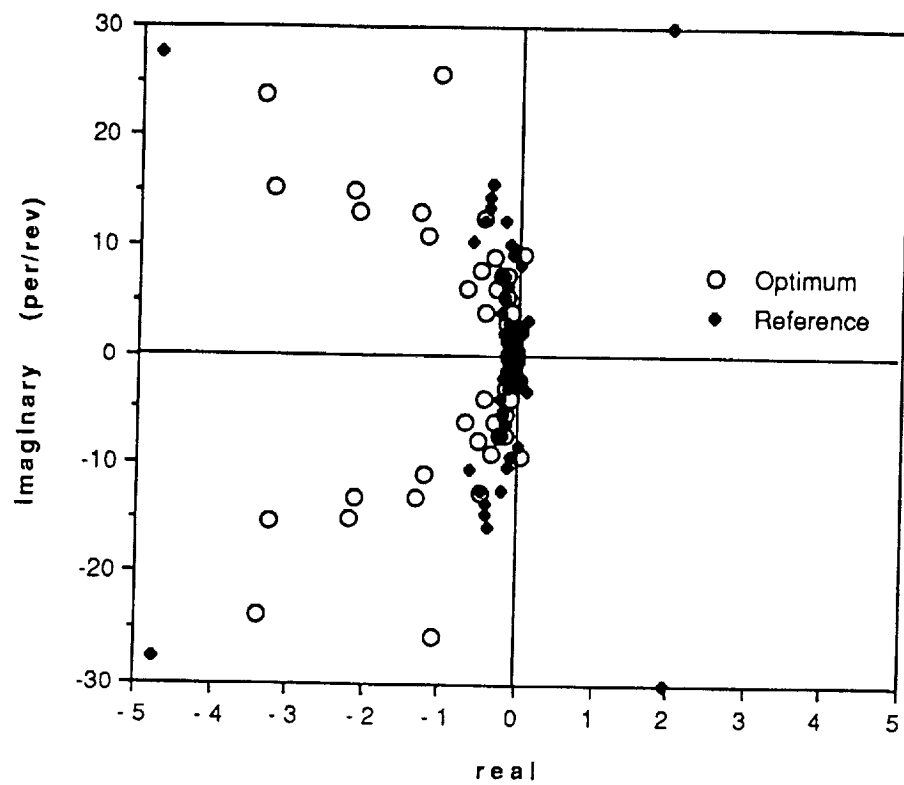


Figure 3 Stability roots in hover

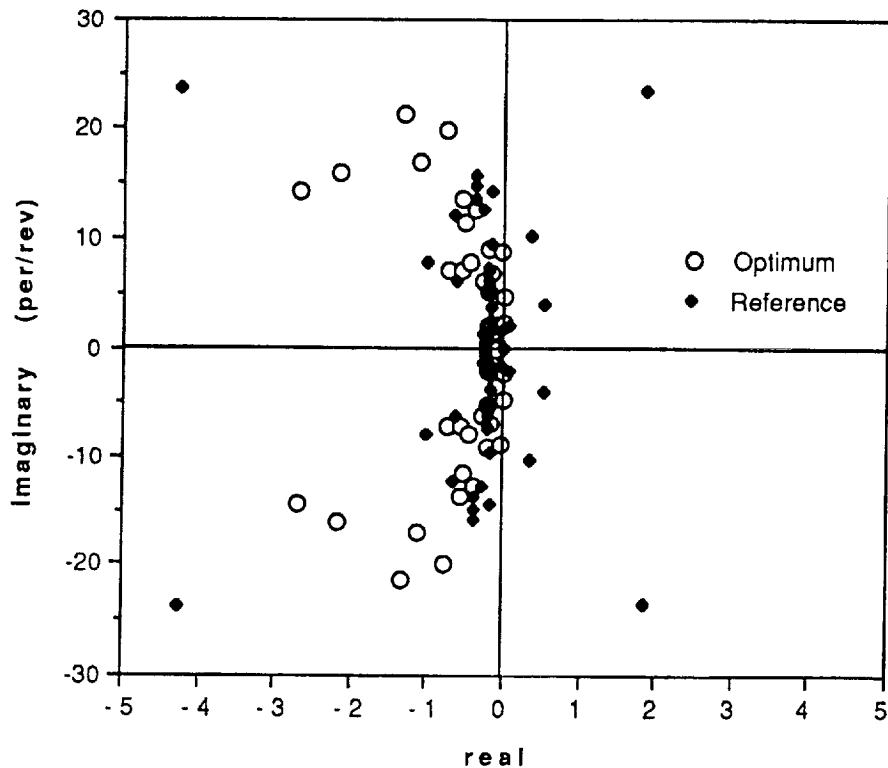


Figure 4 Stability roots in cruise

### Acknowledgement

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# Optimum Design of High Speed Prop-Rotors

by

Aditi Chattopadhyay  
Arizona State University  
Tempe, Arizona 85287-6106

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## Objectives

The objective of this research is to develop optimization procedures to provide design trends in high speed prop-rotors. The necessary disciplinary couplings are all considered within a closed loop optimization process. The procedures involve the consideration of blade aeroelastic, aerodynamic performance, structural and dynamic design requirements. Further, since the design involves consideration of several different objectives, multiobjective function formulation techniques are developed.

## Accomplishments

### Multiobjective Formulation

This goal of the first part of this study is to develop multiobjective optimization formulation procedures for application in multidisciplinary design problems. Due to the fact that some of the optimization problems involve more than one design objective, the objective function formulation is more complicated. In most of the existing work, the individual objective functions are combined using weight factors in a linear fashion. Such methods are judgmental as the answer depends upon the weight factors which are often hard to justify. Two multiobjective function formulation techniques have been investigated and implemented. They are the Minimum Sum  $\beta$  (Min  $\Sigma\beta$ ) [1] and the Kreisselmeier-Steinhauser (K-S) function [2] approaches.

### Optimization Implementation

A nonlinear programming method, as implemented in the numerical code CONMIN [3], is used for the optimization. CONMIN uses the method of feasible directions. In the optimization process, many evaluations of the objective function and constraints are required before

convergence to an optimum design is obtained. Therefore, the process can become computationally expensive if exact analyses are performed for every function evaluation. Therefore the use of an approximate analysis is implemented in the calculations of both the objective functions and the constraints. The approximate analysis used for this study is the two point exponential procedure developed by Fadel et al. [4]

### Optimization Problems

Several multidisciplinary optimization problems involving the coupling of necessary disciplines that are crucial for tilting prop-rotor design have been developed. For example, the propulsive efficiency in high speed cruise (400 knots) and the hover figure of merit have been simultaneously maximized using both the Min  $\Sigma\beta$  and K-S function approaches. Constraints are imposed on the first natural frequency in hover and on the total blade weight. Both aerodynamic and structural design variables are used [5,6]. Next, an integrated structural and aeroelastic optimization procedure using a composite box beam model as the structural load carrying member has also been accomplished [7]. Other work that has been completed includes a minimum rotor drive system weight optimization and a minimum blade weight optimization.

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